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Sequential Sampling Inspection and Cost-profit Balancing Decision Scheme for Multi-stage Electronic Assembly Production

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Abstract

This study targets the persistent supply chain defect rate problem occurring in manufacturing production workflows and develops an integrated quality control and decision optimization framework built upon binomial-distribution hypothesis testing, multi-objective goal programming, and Bayesian statistical inference. To start, simple random sampling and sequential sampling strategies are comparatively integrated into a Binomial Distribution inverse cumulative distribution function (Binomial-ICDF) hypothesis testing model, which calculates the minimum sampling volume and critical defect thresholds corresponding to varied confidence levels. Comparative experimental results verify the higher operational efficiency of sequential sampling. Specifically, under the 95% confidence level, the minimum sample size is 98, and batches with five or more defective products are rejected; when the confidence level drops to 90%, the required sample size decreases to 60, and batches with fewer defective items than the threshold of nine are qualified for acceptance. On this basis, a bi-objective goal programming model is established that comprehensively incorporates multi-dimensional cost indicators including procurement expenditure, assembly outlay, inspection fees, product replacement costs and disassembly expenses, so as to optimize combined quality control schemes and overall cost performance under diverse actual defect rate scenarios. Furthermore, Monte Carlo random sampling simulation is adopted to generate stochastic production defect samples, combined with Bayesian posterior updating to dynamically calibrate the probability distribution of batch defect rates. Scenario-based sensitivity analysis reveals that despite observable fluctuations in corporate profit margins across simulation cases, the derived optimal quality control decision scheme maintains strong stability. Industrial verification demonstrates that the proposed integrated model can simultaneously lift production operational efficiency and corporate profit levels while mitigating profit volatility risks, which validates its robust adaptability and practical application value for on-site manufacturing supply chain quality governance.

Keywords: Binomial Distribution-ICDF Model, Dual-Objective Programming, Bayesian Statistics, Monte Carlo Simulation

Introduction

With the deepening development of globalization and international trade, economic linkages and mutual dependence among countries have been continuously strengthened. To stand out in fierce market competition, enterprises must continuously enhance their core competitiveness and seize market opportunities. In this process, effective supply chain management has become an important means of improving competitive advantage and customer satisfaction. However, the complexity of supply chains also brings greater uncertainty and potential risks [1]. Data show that the quality of components during production directly affects the overall quality and market competitiveness of finished products. Facing growing market demand and limited resources, traditional production methods tend to lead to rising defect rates, seriously affecting the economic benefits of enterprises.

In practice, enterprises have a variety of sustainable operation strategies to choose from at different stages of the product lifecycle, and different enterprises often adopt different strategies based on their own conditions [2]. Therefore, how to reduce defect rates and improve efficiency through optimizing production decisions has become an urgent issue to be addressed.

At present, a large number of studies have proposed various algorithms for quality control problems in production processes, such as linear programming and dynamic programming. In recent years, with advances in computational technology, emerging methods such as genetic algorithms and neural networks have also been gradually applied to production decision optimization. Although these algorithms can provide solutions quickly, they still suffer from insufficient adaptability in certain cases, suggesting that optimization models more suitable for actual production environments should be developed.

1. Hypothesis Testing Model Based on Binomial Distribution–ICDF

Hypothesis testing is a statistical research method that uses the principle of small probability to infer, through random sample information, whether a pre-made assertion about a certain quantitative characteristic of a population is valid [3]. This study compares simple random sampling and sequential sampling, and comprehensively considers decision-making. It is known that Hubei Ding sheng Electronics Co., Ltd. plans to purchase a batch of components (Component 1 or Component 2) from a supplier. Regarding whether the defect rate exceeds a certain nominal value, two cases are considered: Case 1: at a 95% confidence level, if the component defect rate is determined to exceed the nominal value, the batch is rejected; Case 2: at a 90% confidence level, if the component defect rate is determined not to exceed the nominal value, the batch is accepted.

1.1 Simple Random Sampling

1). Establishing Hypotheses

Null hypothesis (H0): For Case 1, the defect rate exceeds the nominal value; for Case 2, the defect rate does not exceed the nominal value.

Alternative hypothesis (H1): For Case 1, the defect rate does not exceed the nominal value; for Case 2, the defect rate exceeds the nominal value.

2). Selecting Significance Level and Testing Method

When the confidence level is 95%, the significance level α = 0.05; when the confidence level is 90%, α=0.1. In this problem, according to the Central Limit Theorem, when the sample size is sufficiently large, the binomial distribution can be well approximated by the normal distribution. Therefore, the normal approximation is chosen in this case.

3). Calculating Sample Size and the Number of Defects Corresponding to the Critical Value

The calculation of sample size and the corresponding number of defects ensures the effectiveness of the sampling inspection plan. To minimize the number of inspections while meeting a given confidence level (e.g., 95% or 90%), it is necessary to determine the minimum sample size that achieves the required detection accuracy. This involves constructing and solving an inequality regarding sample size, which takes into account the defect rate, confidence interval, and the required statistical power (i.e., the probability of correctly accepting or rejecting the null hypothesis). Solving this inequality ensures that, under the given defect rate, there is sufficient confidence to decide whether to reject the null hypothesis.

4). Calculating the Test Statistic

The sample defect rate is used as an unbiased estimate of the population defect rate, and its standard error is further calculated. The test statistic is the difference between the sample defect rate and the nominal defect rate, divided by the standard error, yielding the test statistic Z. And Z reflects the degree of deviation of the sample defect rate from the nominal defect rate and is used for subsequent hypothesis testing decisions.

$$Z=\frac{\text{Sample defect rate}-\text{Nominal defect rate}}{\text{standard error}} \tag{1}$$

5). Judging and Drawing Conclusions

Based on the calculated results and decision criteria, determine whether there is sufficient evidence to reject the null hypothesis.

1.2 Sequential Sampling

Sequential sampling is a dynamic sampling method in which the sample size is not fixed in advance but is determined progressively based on the sampling results.

1). Establishing Hypotheses

Null hypothesis (H0): For Case 1, reject when the cumulative number of defects reaches a certain threshold; for Case 2, accept when the cumulative number of defects does not reach a certain threshold.

Alternative hypothesis (H1): For Case 1, accept when the cumulative number of defects does not reach a certain threshold;

for Case 2, reject when the cumulative number of defects reaches a certain threshold.

2). Determining a Stopping Rule and Threshold

This is typically achieved through Sequential Probability Ratio Testing (SPRT). However, for simplification, a simple rule can be used, for example: if more than 11% defects are found in the first k samples, stop sampling and reject; if less than 11% defects are found in the first k samples, continue sampling.

3). Calculating the Sample Size and the Actual Number of Defects Corresponding to the Critical Value

The specific minimum number of inspections needs to be determined through simulation or calculation, but it is usually smaller than that of simple random sampling.

4). Judging and Drawing Conclusions

Based on the calculated results and decision criteria, determine whether there is sufficient evidence to reject the null hypothesis.

1.3 Comparison of Results

Sequential sampling is superior in terms of the number of inspections because it allows stopping sampling once the decision criterion is met. However, the formulation and implementation of a sequential sampling plan are generally more complex than simple random sampling. Enterprises should choose the appropriate plan based on their resources, risk tolerance, and operational convenience. In most cases, sequential sampling would be the more economical choice.

1.4 Establishment of the Hypothesis Testing Model Based on Binomial Distribution–ICDF

1. **Determine the Null Hypothesis H_0 and Alternative Hypothesis H_1.**

   H_0: Component defect rate $p \leq 0.10$;

   H_1: Component defect rate $p > 0.10$.

2. **Choose significance levels** $\alpha = 0.1$ and $\alpha = 0.05$, and look up the values of $Z_{0.05}$ and $Z_{0.10}$ for one-sided tests from the standard normal distribution table.

3. **Calculate the Sample size n.**

$$n = \frac{(Z_{alpha}{}^2) \cdot p_0 \cdot (1-p_0)}{E^2} \tag{2}$$

where Z_{alpha} is the upper alpha quantile of the standard normal distribution (If alpha=0.05, then $Z_{0.05} \approx 1.645$; If alpha = 0.10, then $Z_{0.10} \approx 1.282$); $P_0 = 0.1$ is the upper limit of the defect rate claimed by the supplier; E=0.05is the error margin.

4. **Calculate the Critical Number of Defects Xc.**

$$X_c = p_0 \cdot n + Z_{alpha} \cdot \sqrt{p_0 \cdot (1-p_0) \cdot n} \tag{3}$$

5. **Let the number of defects in the sample be Xc, and the sample defect rate be p^.**

According to the Central Limit Theorem [4], p^ approximately follows a normal distribution $N(p, \frac{p(1-p)}{})$. Calculate Z:

$$\hat{p} = \frac{X}{n} \tag{4}$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \tag{5}$$

6. **Compare Z** with Z_{alpha}. If $Z \geq Z_{alpha}$, then reject the null hypothesis; if $Z < Z_{alpha}$, Then accept the null hypothesis.

1.4.1 Solution for Case 1

Assume that the component defect rate provided by the supplier exceeds the nominal value (10%), and a right-tailed test is conducted at significance level α=0.05 (95% confidence), with $Z_{0.05}$=1.645. The model results for this case are shown in Figure 1.

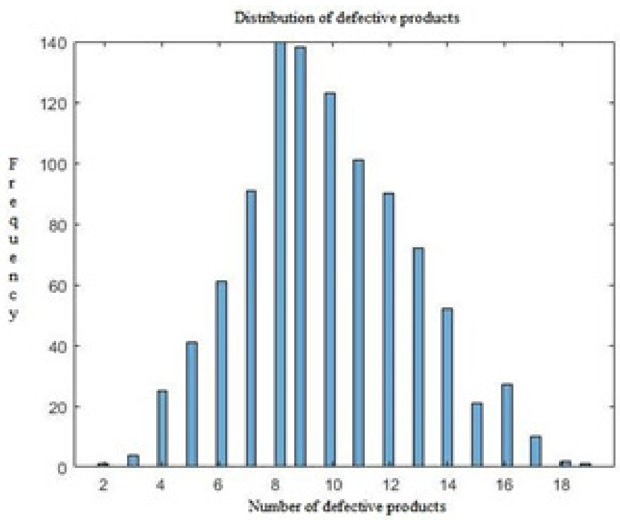

Figure 1: Defect Quantity Distribution for Case 1.

The enterprise needs to draw 98 component samples for inspection to ensure effective decision-making at a 95% confidence level; the critical defect quantity Xc=5; the test statistic Z=16.2832. Since $Z > Z_{0.05}$, at a 95% confidence level, if the number of defective samples is greater than or equal to 5, the null hypothesis is rejected, and the batch is rejected.

1.4.2 Solution for Case 2

Assume that the defect rate does not exceed the nominal value. The enterprise conducts testing at a 90% confidence level and makes decisions at a significance level α=0.10 (90% confidence), with $Z_{0.10}$ = 1.282 . The model results are shown in Figure 2.

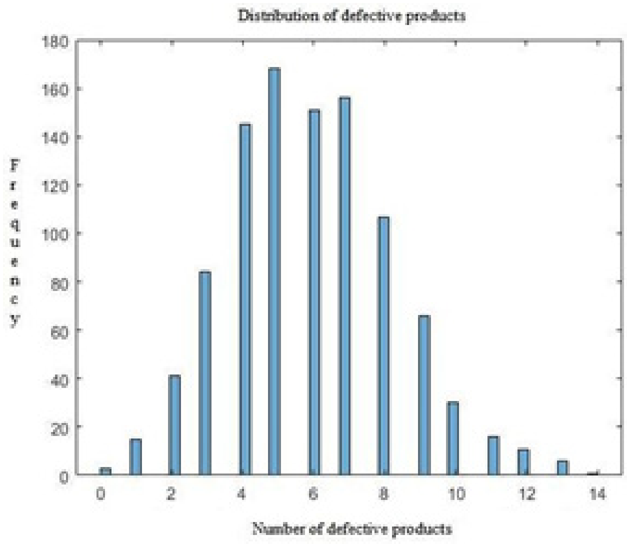

Figure 2: Defect Quantity Distribution for Case 2.

The enterprise needs to draw 60 component samples for inspection to make a decision at a 90% confidence level; the critical defect quantity Xc = 5; the test statistic Z = 9.9269. Since Z >$Z_{0.10}$, at a 90% confidence level, if the number of defective samples is less than 9, the null hypothesis is accepted, and the batch is accepted.

1.4.3 Result Analysis

By designing a sampling plan with the minimum number of inspections, enterprises can effectively judge the component defect rate with as few inspections as possible while ensuring sufficient confidence, thereby making reasonable procurement decisions.

1.4.4 Sensitivity Analysis

To evaluate the sensitivity of model outputs to changes in input parameters. In quality control and supply chain management, understanding the sensitivity of the model to key parameters such as the error tolerance (E) is particularly important because this parameter may affect the decision-making process and final results. Through sensitivity analysis, the impact of this parameter on decisions can be identified, so that in practice, more attention can be paid to the control and optimization of these parameters.

The following figures show the effects of error tolerance on various parameters in Case 2. Figure 3 shows the effect of error tolerance on sample size, Figure 4 shows the effect of error tolerance on defect quantity.

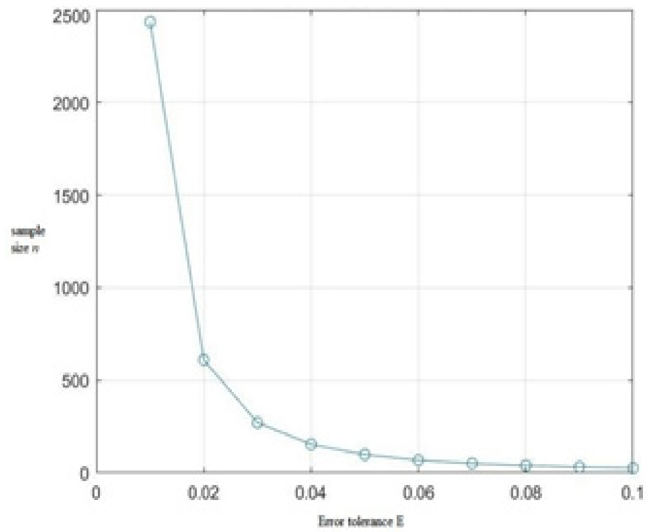

Figure 3: Effect of error tolerance on sample size.

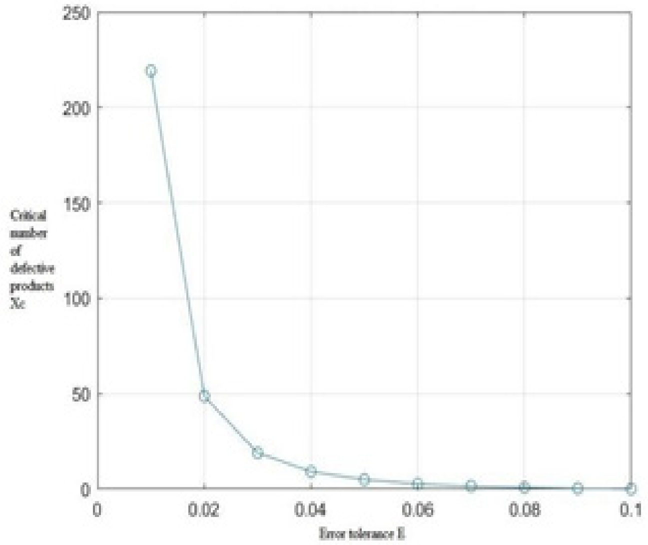

Figure 4: Effect of Error Tolerance on Defect Quantity.

Through the above sensitivity analysis, it is determined that E has a significant effect on sample size and defect quantity, but has a relatively small effect on the Z statistic. The relationship between the magnitudes of Z and Zalpha does not change, indicating that the hypothesis testing model is relatively stable, and the error tolerance does not affect the final decision of acceptance or rejection.

For enterprises, in practice, more attention should be paid to this parameter. By understanding the impact of changes in E on decisions, the decision-making process can be optimized, and the risk of erroneous decisions can be reduced.

2. Stage Division

After deciding to purchase this batch of components, the enterprise needs to proceed with finished product assembly. The entire production process involves the following steps, including inspection, assembly, disassembly, etc.

2.1 Component Inspection Stage

In the production process shown in Figure 6, the enterprise first needs to purchase two types of components (Component 1 and Component 2) for finished product assembly. The key decision at this stage is whether to inspect the purchased components to ensure that defective items do not enter subsequent assembly steps. The enterprise must balance component procurement costs, inspection costs, and the risk of defects entering production. Trade-off between inspection and non-inspection:

1. **Choosing Inspection:** Inspecting components incurs inspection costs d1 and d2 , but can eliminate nonconforming components and prevent defects from affecting finished product quality. This helps reduce subsequent costs for handling nonconforming finished products.
2. **Choosing Non-Inspection:** Not inspecting saves inspection costs but may allow defective components to enter the assembly stage, thereby increasing the finished product defect rate. This increases subsequent product handling costs and replacement losses, affecting the enterprise's profit. Therefore, when inspection costs are low or component defect rates are high, inspection is the better choice; however, if inspection costs are too high and defect rates are low, non-inspection is more economical.

2.2 Finished Product Inspection Stage

The production of finished products depends on qualified components. Whether the assembled finished products meet market quality requirements is key to the enterprise's profitability. Therefore, whether to inspect finished products is another important decision point. Trade-off between inspection and non-inspection:

1. **Choosing to Inspect Finished Products:** Finished products have a high market price. Inspecting finished products can reduce the risk of nonconforming products entering the market and lower subsequent replacement losses (logistics costs, damage to corporate reputation, etc.).
2. **Choosing not to Inspect Finished Products:** If the finished product defect rate is low or replacement losses are small, the enterprise may choose not to inspect finished products and directly send them to market. However, when defect rates are high or the market is sensitive, this approach may incur greater long-term costs.

2.3 Disassembly and Reflow Stage

After inspecting finished products, the enterprise may discover some nonconforming products. At this point, it needs to decide whether to disassemble them for recycling and reuse of components. This decision depends on disassembly costs and potential recycling value, as well as the replacement losses that nonconforming products might cause if they enter the market. Trade-off between disassembly and scrapping:

1. **Choosing disassembly:** Disassembling nonconforming finished products can recover some components, reduce waste, and lower negative environmental impact. Although disassembly incurs additional cost C_4 (disassembly cost), the recovered components can be used in subsequent production, reducing overall production costs. When disassembly costs are low and there are many nonconforming finished products, disassembly is the more economical choice.
2. **Choosing scrapping:** If disassembly costs are high, or if the component defect rate is high and components have little recycling value, the enterprise may choose to directly scrap the nonconforming finished products, saving disassembly costs.

3. Establishment and Solution of the Goal Programming Model

3.1 Model Establishment

Objective functions are constructed for two cases depending on whether finished products are inspected [5]. Assume that non-inspection status of finished products is 0, inspection status is 1; finished product disassembly status is 1, non-disassembly status is 0; Component 1 inspection status is 1, non-inspection status is 0; Component 2 inspection status is 1, non-inspection status is 0.

1). Finished products not inspected

If finished products are not inspected, replacement will occur in the subsequent process. According to permutation and combination, there are eight cases, as shown in Table 1.

Table 1: Eight States when Finished Products are not Inspected

Component 1 Status	Component 2 Status	Finished Product Inspection Status	Finished Product Disassembly Status
0	0	0	1
0	1	0	1
1	0	0	1
1	1	0	1
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	0

Constraints: Assume the data for the situations encountered by the enterprise in production are shown in Table 2.

When finished products are not inspected, from Table 2 we can derive $P_{\text{defect rate of component}}$, the total purchase unit price of components c_{total}, the total inspection cost of components d_{Totel}, and the total defect probability $P_{\text{defect probability}}$.

Then the objective function, i.e., profit T is:

$$\max T = s\cos\cdot\left(p_{\text{defect rate of component}}\right) - \left[c_e + d_e - \left(rloss + dis\cos t - (c_1 + c_2)\right)\cdot p_{\text{defect probability}}\right] \tag{6}$$

Where

s cos t denotes the market selling price of finished products, and rloss denotes the replacement lossfornonconforming finished products.

Table 2: **Situations Encountered by the Enterprise in Production**

Situation	Component 1			Component 2			Finished Product			Nonconforming Finished Product		
	Defect Rate	Unit Price	Insp. Cost	Defect Rate	Unit Price	Insp. Cost	Defect Rate	Assembly cost	Insp. Cost		Exchange losses	
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	10%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

Finished Products Inspected

If finished products are inspected, disassembly and non-disassembly may occur in the subsequent process. According to permutation and combination, there are eight cases, as shown in Table 3.

Table 3: Eight States When Finished Products are Inspected

Component 1 Status	Component 2 Status	Finished Product Inspection Status	Finished Product Disassembly Status
0	0	1	1
0	1	1	1
1	0	1	1
1	1	1	1
0	0	1	0
0	1	1	0
1	0	1	0
1	1	1	0

When finished products are inspected, the objective function, i.e., profit T is: (max = cos).

$$[To\ pTT\ oe\ p - + - \ (\cos - 1 + 2 \cdot To\ pT\ oep)] \quad (7)$$

The constraints are the same as the data in Table 2.

3.2 Model Solution

Since there are many subdivided cases and the calculation methods are similar in each case, the case where neither Component 1, Component 2, nor finished products are inspected is taken as an example for solution.

In this case, the total cost is:

$$total\cos t = c_1 + c_2 + c_3 + rloss(p_1 + p_2 + (1-p_1)\cdot(1-p_2)\cdot p_3) \tag{8}$$

Total revenue is:

$$totalrevenue = s\cos t \cdot (1-(p_1 + p_2 + (1-p_1)\cdot(1-p_2)\cdot p_3)) \tag{9}$$

Total profit is:

$$T = totalrevenue - total\cos t \tag{10}$$

Since replacement and disassembly of finished products can only occur when finished products are nonconforming, when calculating the qualified rate, only the eight cases of whether Components 1, 2 and finished products are inspected are considered.

Based on the defined objective functions, MATLAB programming was used to solve for the profit margin R. The detailed results for the six situations are shown in Table 4.

Table 4: Optimal schemes under six situations

Situation	Decision Scheme	Profit Margin
Case 1	1101	63.94%
Case 2	1101	35.49%
Case 3	1101 (1111)	47.85%
Case 4	1111	63.12%
Case 5	0011	71.34%

In summary, under various production environments, when component defect rates are high or finished product replacement costs are large, inspection is an effective method to reduce subsequent costs and increase profit. In addition, for nonconforming finished products, disassembly is generally used to recover some costs, but when disassembly costs are too high, directly disposing of these nonconforming products may be more economical. The final decision schemes for the six situations are 1101, 1101, 1101 (1111), 1111, 0011, and 0000, with profit margins of 63.94%, 35.49%, 47.85%, 63.12%, 71.34%, and 73.46%, respectively.

4 Designing Decision Schemes

4.1 Model Preparation

It is now known that the enterprise's production process involves m processes and n components, and the defect rates are distributed across components, semi-finished products, and finished products, with data shown in Table 5. Therefore, a new goal programming model needs to be constructed to model costs and benefits at each stage, thereby obtaining the optimal combination of inspection/ disassembly decisions in the production process. Table 5 Defect rate distribution across stages

Component	Defect Rate	Unit Price	Insp. Cost	Semi-finished Product	Defect Rate	Assembly Cost	Disassembly Insp. Cost	Cost
1	10%	2	1	1	10%	8	4	6
2	10%	8	1	2	10%	8	4	6
3	10%	12	2	3	10%	8	4	6
4	10%	2	1					
5	10%	8	1	Finished Product	10%	8	6	10
6	10%	12	2					
7	10%	8	1		Market Price		Replacement Loss	
8	10%	12	2	Finished Product	200		40	

4.1.1 Decision Variables

1) Component inspection decision variables

$$decision.vector(i), i = 1, \cdots, n \tag{11}$$

Value 1 indicates inspection; 0 indicates non-inspection.

2) Semi-finished Product Inspection Decision Variables

$$decision.vector(j+n), j = 1, \cdots, m \tag{12}$$

3) Finished Product Inspection Decision Variable

$$decisim.vector(2m+n+1) \tag{13}$$

Value 1 indicates inspection; 0 indicates non-inspection.

4) Semi-finished Product Disassembly Decision Variables

$$decisim.vector(n+m+j), j=1,\cdots,m \tag{14}$$

5) Finished Product Disassembly Decision Variable

$$decisim.vector(2m+n+j) \tag{15}$$

Value 1 indicates disassembly; 0 indicates non-disassembly. And a total of sixteen decision variables are considered.

4.1.2 Production Stages

There are interrelationships among components, semi-finished products, and finished products. The defect rate of the previous stage directly affects cost accounting in the subsequent stage, and the defect rate throughout the entire production process has a cumulative impact on costs.

1). Component Stage

The quantity of qualified components N.component. qualified (i) :

$$N.component.qualified(i) = \begin{cases} N.component(i) \cdot (1 - component.defectrate(i)) \\ \text{Not detecting parts} \\ N.component(i) \\ Testing\ parts \end{cases} \tag{16}$$

2). Semi-Finished Product Stage

The defect rate of semi-finished product semi.defect.rate(j) :

$$semi.defect.rate(j) = 1 - \prod (1 - component.defect.rate(i)) \tag{17}$$

3) Finished Product Stage

The defect rate of finished product product.defect.rate:

$$product.defect.rate = 1 - \prod (1 - semi.defect.rate(j)) \tag{18}$$

The quantity of qualified finished products N.product.qualified:

$$N.product.qualified = (1 - product.defect.rate) \cdot N.product \tag{19}$$

where N.product is the total quantity of finished products.

4.2 Model Establishment

4.2.1 Costs

1) Purchase Cost

$$C.total = \sum_{i=1}^{n} N.component(i) \cdot component.cost(i) \tag{20}$$

Among them, C.total is the total purchase cost of the parts; N.component(i) is the purchase quantity of part i component.cost(i) is the purchase unit price of part i

2) **Total cost** total.cost.

$$total.\cos t = c.total + d.total + e.total + f.total + g.total \tag{21}$$

4.2.2 Revenue

$$total.revenue = N.product.qualified \cdot product.market.price \tag{22}$$

Among them, *N.product.qualified* refers to the qualified quantity of finished products; *product.market. price* is the market price of the finished product. 4.2.3 Profit

4.2.3 Profit

$$\max total.profit = total.revenue - total.\cos t \tag{23}$$

Finally:

$$\begin{aligned} \max total.profit = N.product.qualified \cdot product.marke t. price \\ -(c.total + d.total + e.total + f.total + g.total) \end{aligned} \tag{24}$$

Maximize profit by optimizing the decision variables decision.vector(i).

Constraints: Composed of the sixteen decision variables and the data in **Table 5**.

4.3 Model Solution

Finally, the optimal combination of inspection/disassembly decisions is: 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1; the maximum profit margin is 34.68%.

5. Re-estimation of Defect Rates

Bayesian statistical methods and Monte Carlo simulation are used to re-estimate the supplier's defect rate. This involves two main steps: Monte Carlo simulation is used to generate random samples to estimate the number of defects, and Bayesian updating uses the prior distribution and defect inspection results to update the posterior distribution of the defect rate. The uncertainty of sampling inspection needs to be considered in the sampling decision process.

1). **Parameter Setting and Sample Size Calculation**

Parameter setting

α = 0.05 : significance level, which representing the probability of committing a Type I error in statistical testing; p_0 = 0.10: the upper limit of the defect rate claimed by the supplier; α_{prior}=1， β_{prior}=1: prior parameters of the Bayesian distribution, which representing an initial uniform assumption (no particular preference) about the defect rate.

2). Calculation of Sample Size *n*

To estimate the defect rate at a given confidence level, the following formula is used to calculate the sample size:

$$n = \frac{Z_{1-\alpha}^2 \cdot p_o \cdot (1-p_0)}{E^2} \tag{25}$$

where $Z_{1-\alpha}$ is the $1-\alpha$ quantile of the standard normal distribution（If $\alpha=0.05$, then $Z_{0.95}\approx 1.645$）; $p_0=0.1$ is the upper limit of the defect rate claimed by the supplier; $E=0.05$ is the error margin.

Therefore, the sample size n is calculated as:

$$n=\frac{(1.645)^2\cdot 0.1\cdot(1-0.1)}{0.05^2} \tag{26}$$

Monte Carlo Simulation

Monte Carlo simulation is used to generate random samples multiple times to estimate the number of defects. In each simulation, several random samples are generated and judged as defective or not. The randomly generated *samples* satisfies a uniform distribution $U(0,1)$, and the following inequality is used to determine whether it is defective:

$$samples_j = 1(rand_j < p_0) \tag{27}$$

Where the right side is an indicator function, indicating that the sample is defective when the random number is less than the defect rate, with a value of 1; otherwise 0[6].

After each simulation, the number of defects k_i is calculated:

$$k_i=\sum_{n}^{j-1} 1(rand_j < p_0) \tag{28}$$

A total of 10,000 simulations are performed, yielding $k_1, k_2, \cdots, k_{10000}$ such defect counts.

5.3 Bayesian Updating

Bayesian statistics is based on the updated posterior distribution. Assuming the defect rate p follows a Bayesian distribution with a prior distribution $Beta(\alpha_{prior}, \beta_{prior})$, then based on the number of defects k_i from each simulation, the updated posterior distribution is:

$$\alpha_{post}=\alpha_{prior}+k_i \tag{29}$$

$$\beta_{post}=\beta_{prior}+(n-k_i) \tag{30}$$

where k_i is the number of defects, and $(n-k_i)$ is the number of non-defective items.

5.4 Mean of the Posterior Distribution as the Estimate of the Defect Rate

The mean of the posterior distribution $Beta(\alpha_{prior}, \beta_{prior})$ provides the estimate of the defect rate:

$$p_{hat}=\frac{\alpha_{post}}{\alpha_{post}+\beta_{post}} \tag{31}$$

This value is the updated estimate of the defect rate based on the defect counts obtained from simulation.

5.5 Re-analysis and Solution of the Goal Programming Model

Since sampling inspection may introduce sampling errors, when evaluating defect rates of components and finished products, it is necessary to comprehensively consider inspection costs, sampling errors, and risk tolerance to formulate reasonable inspection decisions [7].

1) Specific Decision Bases and Indicators

Total cost: Includes inspection costs, disassembly costs, and potential replacement losses. Reasonable inspection decisions help reduce future high costs; Nonconformance loss: financial losses estimated based on defect rates, especially replacement and market outflow risks of defective products; Sampling error: sampling inspection has certain errors; Balance between cost and risk: If inspection cost is lower than loss, increase inspection frequency or perform full inspection. If risk is tolerable, reduce inspection frequency to lower production costs, but accept potential replacement losses.

2) Specific Decision Schemes are shown in Table 6.

Table 6: Optimal Schemes Under Six Situations (Re-Estimated)

Situation	Decision Scheme	Profit Margin
Case 1	1101	41.36%
Case 2	1101	15.43%
Case 3	1101 (1111)	29.29%
Case 4	1111	44.59%
Case 5	0011	52.47%
Case 6	0000	55.51%

5.6 Re-analysis and Solution of the Designed Decision Scheme

For the designed decision scheme, we also need to consider the impact of defect rate uncertainty on production process decisions. Since multiple components and multiple processes are involved, the impact of uncertainty may be more complex.

1). Decision scheme

Component inspection: Given that the defect rates of all components are 10% and inspection costs are relatively low, inspection is recommended to reduce disassembly costs; Semi-finished product inspection: For semi-finished products 1, 2, and 3, given a defect rate of 10% and low inspection costs (inspection cost lower than disassembly cost), inspection is recommended; Semi-finished product disassembly: For semi-finished products 1, 2, and 3, given a defect rate of 10% and relatively high disassembly costs, disassembly is not recommended; Finished product inspection: Since the finished product defect rate is 10% and inspection costs are relatively high, while replacement costs are relatively low, it is recommended not to inspect finished products and directly release them to the market as more economical; Finished product disassembly: Since the finished product defect rate is 10% and inspection is not recommended, and considering that disassembly costs for nonconforming finished products are relatively low, disassembly of nonconforming finished products is recommended.

2) Specific decision bases and indicators

Component procurement: Select components with lower costs for production without inspection; Semi-finished product handling: Assemble semi-finished products with a 10% defect rate without inspection; Finished product handling: Directly release finished products to the market to avoid high inspection costs; Cost-benefit analysis: By comparing total costs and revenues of various strategies, this scheme achieves an effective balance between cost and quality, thereby maximizing profit; Risk management: By reducing inspection of key components, production costs are lowered while maintaining the enterprise's brand image and financial stability.

3) Specific decision scheme

Optimal combination of inspection/disassembly decisions: 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1; maximum profit margin: 32.37%.

5.7 Sensitivity Analysis of Re-estimated Defect Rates

In Bayesian statistics, prior parameters have a significant impact on the posterior distribution, so understanding how they affect the final estimation results is important [8]. In the re-estimation of defect rates, the prior parameters α and β as variables may affect the estimated defect rate, causing fluctuations in the defect rate and affecting product quality and revenue. Below, a sensitivity analysis is performed on parameters α and β, where both α and β range from 1 to 5 with a step size of 0.1. The difference between the defect rate when prior parameters change and the original defect rate shows the impact of prior parameter changes on the defect rate.

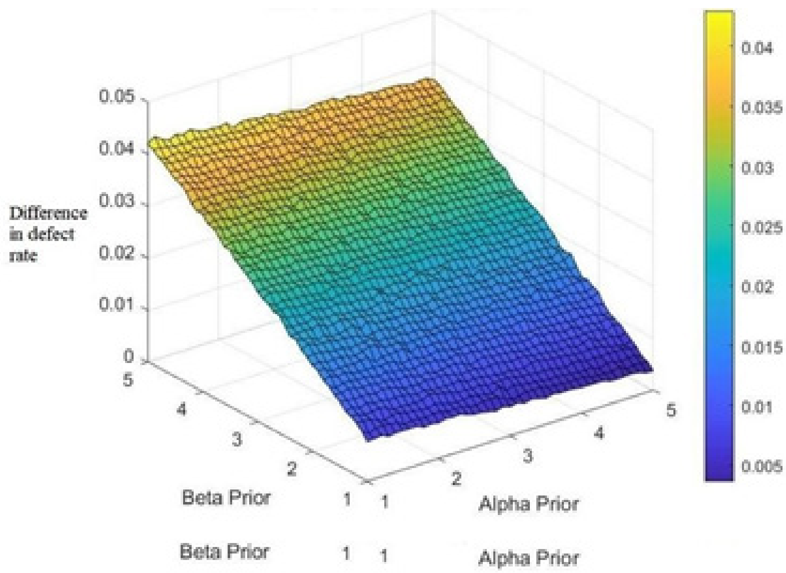

Figure 5: Effect of Prior Parameters on the Defect Rate Difference.

Through the sensitivity analysis of prior parameters a and b, it can be concluded that prior parameters have little effect on the posterior distribution and estimation results. For enterprises, considering and analyzing the effects of prior parameters a and b can optimize the decision-making process and improve decision accuracy and reliability. Through the above sensitivity analysis, we can better understand how the choice of prior parameters affects Bayesian estimation results and make more reasonable decisions accordingly [9].

5.8 Model Advantages and Extensions

First, it fully integrates practical conditions, simplifies the inspection process, defect rate analysis, and multi-process quality control constraints, taking into account factors such as defect rate propagation, inspection costs, and market replacement costs, forming a reasonable model with high application value, which can be extended to multi-stage production and supply chain management. Second, it employs Bayesian statistics and Monte Carlo simulation, capturing key factors in defect rate assessment and decision optimization, transforming complex supplier defect rate evaluation problems into simple probability assessment and cost control problems. With reasonable parameter settings, the output results meet the requirements of the problem and solve the uncertainty in practical decision-making. Third, the Monte Carlo simulation algorithm used has the advantages of high efficiency, stability, and the ability to handle complexity through random sampling, making it suitable for defect rate optimization models in multi-stage production processes. Fourth, the resulting decision strategies are characterized by high efficiency, strong stability, and balanced cost-effectiveness, with essentially no issues such as defect accumulation or excessive inspection, effectively improving production efficiency and financial stability under existing conditions [10].

It can be extended to other similar scenarios, such as supply chain management and multi-stage production processes. By adjusting parameters related to defect rates and inspection costs, quality control and cost optimization problems in other industries can be addressed. Since the production process of electronic products is very complex and involves many influencing factors, it is necessary to comprehensively monitor production conditions, improve existing quality management and control methods, ensure that electronic product quality meets specified requirements, and better facilitate people's daily lives. In conjunction with relevant literature, further consideration of the impacts of market demand fluctuations and production capacity constraints can optimize the model's applicability and enhance its application value in uncertain environments [11].

5. Analysis and Conclusion

1. **Comparison of Sampling Plans:** Based on the binomial distribution ICDF hypothesis test, sequential sampling has higher efficiency, with a 95% confidence level sample size of 98 and a critical defect of 5, and a 90% confidence level sample size of 60 and a critical defect of 9. It can quickly determine batch quality, but the process is complex and suitable for high-value, high quality control required components; Simple random sampling is easy to operate and suitable for low-cost small batch spare parts. Enterprises can choose sampling methods based on their own resources and risk preferences.
2. **Cost benefit of multi-stage inspection:** Production includes three major decision-making nodes: spare parts inspection, finished product inspection, and defective product disassembly. When the defect rate of spare parts is high and the after-sales compensation is large, pre testing can compress the overall cost; If the testing cost is high and the defect rate is low, there is no need to randomly inspect spare parts. Finished product testing can avoid market compensation and brand loss, only low-priced and low after-sales loss products can be exempted from inspection; Disassembly and recycling are only cost-effective when the dismantling cost is lower than the replacement cost of the parts, otherwise it is more economical to directly scrap the defective products.
3. **The optimal detection scheme has scene differences:** six production scenarios correspond to different 0-1 detection combinations. High and low loss scenarios 1-3 select full inspection of spare parts, finished product testing, and dismantling of defective products, with a profit margin of 35.49% to 63.94%; Scenario 4 has the best full process detection, with a profit margin of 63.12%; Scenario 5 only has the highest profit from finished product inspection and disassembly, with a profit margin of 71.34%; Scenario 6 has high testing costs and few defective products, with a profit margin of 73.46% for the entire process without inspection. The multi component complex production line adopts full inspection and selective disassembly, with a stable profit margin of 34.68%.
4. **Application value of the model:** This article integrates hypothesis testing, goal planning, Bayesian updating, and Monte Carlo simulation to quantify the defect rate, cost, and profit of each scheme, changing the traditional empirical quality control model. The model is suitable for electronic assembly enterprises and can be flexibly adjusted according to parameters such as procurement, testing, after-sales, and disassembly, helping enterprises optimize quality control, reduce costs, and stabilize operational efficiency.

Based on the above analysis, it can be concluded that there is no universal optimal solution for production quality control. The core decision-making factors include the defect rate of spare parts, testing, after-sales service, and dismantling costs. Enterprises should quantitatively calculate profits based on their own production data, dynamically adjust sampling, testing, and defect handling

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strategies, and balance quality risks and production costs [12-18].

Conflict of interest

We have no conflict of interests to disclose and the manuscript has been read and approved by all named authors.

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